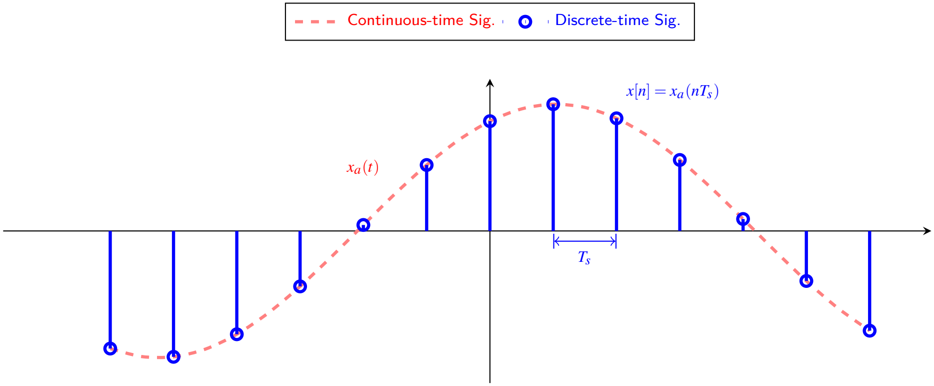
《数字信号处理》：时域中的离散时间信号与系统

Digital Signal Processing: DT Signal and System in Time-domain

Discrete-time Signal

DSP MOOC Course

$$x[n] = x_a(nT_s) : \mathbb{Z} \mapsto \mathbb{R}$$



北卡罗来纳州达勒姆

星期六下午2:00

局部多云

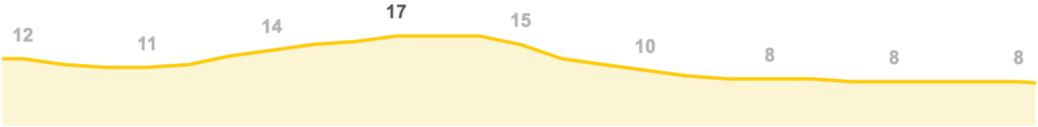
17 °F | °C

降水概率： 1%

湿度： 55%

风速： 5 公里/时

温度 | 降水概率 | 风力

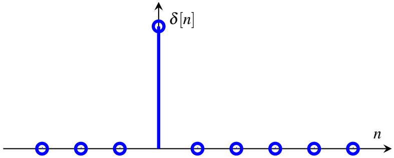


$$\{x[n]\} = \{\cdots, 11, \underset{\underset{x[0]}{\uparrow}}{14}, 17, 15, \cdots\}$$

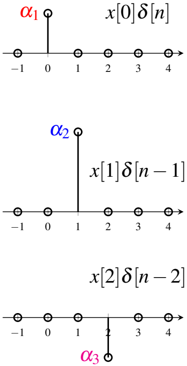
$$\mathbf{x} = \begin{bmatrix} 12 \\ 11 \\ 14 \\ 17 \\ \vdots \\ 8 \end{bmatrix} \in \mathbb{R}^N$$

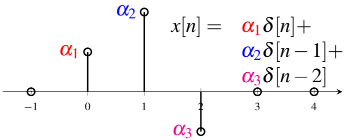
$$\mathbb{R}^N \xrightarrow{N \to \infty} \mathcal{L}^2$$

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

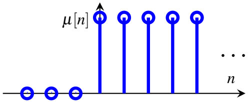


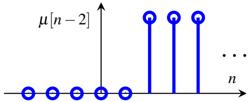


Basic Sequences

- **Dirac (Unit sample) Sequence**
- **Step Sequence**
- **Exponential Decay**
- **Window Sequence**
- **Sinusoid**

$$\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



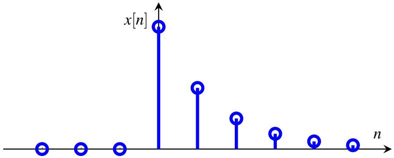


Relation to unit sample sequence

$$\mu[n] = \sum_{k=-\infty}^{n} \delta[k] \qquad \delta[n] = u[n] - u[n-1]$$

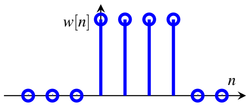
usually exploited to make the sequence causal

$$x[n] = \alpha^n \mu[n], \alpha = 0.5$$



- exponential sequences depicts a large number of physical phenomenon, e.g. *the solution of heat function,*
- it has tight relation to system analysis in next lectures.

$$w[n] = \begin{cases} 1, & N_1 \leq n < N_2 \\ 0, & \text{else} \end{cases}$$

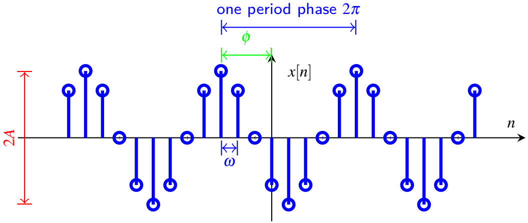


bridges the gap between sequences with infinite and finite length

$$x[n] = w[n] \cdot h[n]$$

then $x[n]$ is cut-off version of $h[n]$ and always with finite length.

$$x[n] = A\cos(\omega n + \phi), \quad -\infty < n < +\infty$$



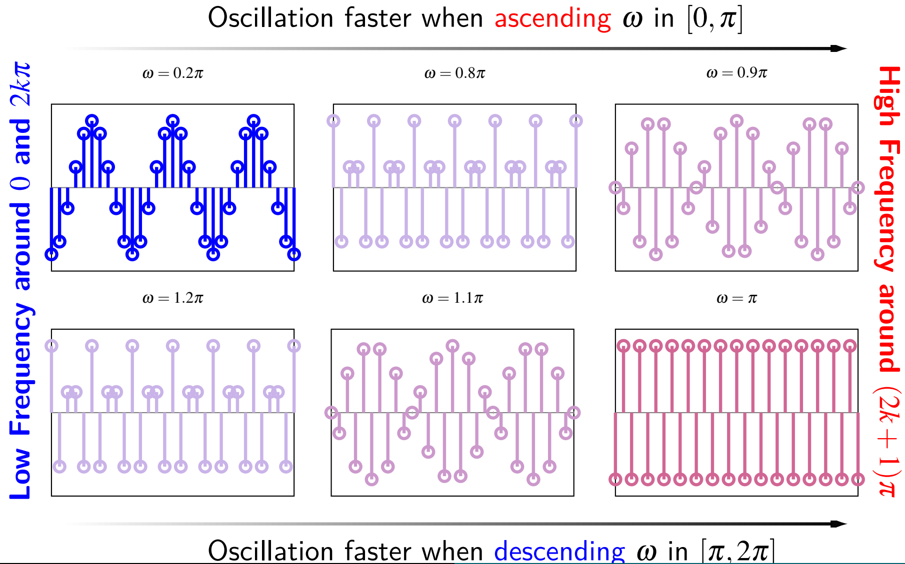
Thus in this example, we can compute

- frequency $\omega = \frac{2\pi}{8} = 0.25\pi$, (rad/sample)
- phase $\phi = 0.75\pi$,
- amplitude A can't be determined.

Define two sinusoids with different frequencies

$$x_1[n] = \cos(\omega_1 n) \quad x_2[n] = \cos(\omega_2 n)$$

when $\omega_2 = \omega_1 + 2\pi k$, x_1 and x_2 are identical


Oscillation faster when ascending ω in $[0, \pi]$
$\omega = 0.2\pi$
$\omega = 0.8\pi$
$\omega = 0.9\pi$
$\omega = 1.2\pi$
$\omega = 1.1\pi$
$\omega = \pi$
Low Frequency around 0 and $2k\pi$
High Frequency around $(2k+1)\pi$
Oscillation faster when descending ω in $[\pi, 2\pi]$

Euler formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$x[n] = A\alpha^n, \qquad A, \alpha \in \mathbb{C}$$